Anybody can do Value at Risk: A Nonparametric Teaching Study

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Abstract

Value at Risk (VaR) has become a benchmark methodology among investors and banks for measuring market risk. Commercially available modelling packages can be both expensive and inflexible, thereby restricting their use by academic researchers and teachers. Using nonparametric methodology, this paper provides a step-by-step teaching study on how to use Excel to construct a VaR spreadsheet for an individual asset as well as for a portfolio. This can benefit financial modelling teachers by providing them with a readily useable teaching study on how to model VaR, as well as benefit researchers by showing them how to construct an inexpensive and flexible VaR model.

Keywords: Financial modelling, value at risk, historical nonparametric value at risk, teaching study, Microsoft Excel.

JEL Classification: C14, C88, G10

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Introduction

Value at Risk (VaR) can be defined as an estimated level of loss on an asset or portfolio for a specified probability (confidence level) and time horizon. The estimate is obtained by measuring variability in rates of return thereby following the tradition of using dispersion of possible outcomes as a measure of risk. A relatively loose distribution of returns suggests higher risk while a tighter distribution suggests lower risk.

Measuring VaR in Finance falls into three general categories: Nonparametric (historical simulation) approach, parametric approach, and Monte Carlo simulation approach (see Culp 2001; Jorion 2001; Linsmeier & Pearson 2000). The essence of parametric methods is that they assume a normal distribution, whereas nonparametric methods make no assumption regarding the distribution. The Monte Carlo method simulates multiple random scenarios. Although VaR is conceptually straightforward, some methodology, particularly Monte Carlo simulation, can be computationally challenging. Of course, VaR calculation can be facilitated by the use of commercially available simulation packages. However, such packages are generally costly and inflexible, allowing the researcher limited scope for adapting the models to their specific requirements. This paper is the first of a series of two papers which demonstrate that the calculation of VaR can be performed using the inexpensive and flexible computer power of Microsoft® Excel, starting with a single asset before proceeding to a portfolio. This paper discusses the use of two VaR nonparametric methods, being firstly the historical method and secondly bootstrapping the historical method, hereafter referred to as the 'historical bootstrap method'. The next paper discusses parametric approaches, including the variance-covariance parametric method and a parametric Monte Carlo approach. As far as we know, besides Day (2003), the calculation of VaR has not been introduced at a significant level in any financial modelling or Excel modelling studies. Our detailed instruction is certainly designed to be far more comprehensive in terms of both concept and algorithm, than any previous instruction, as well as providing a practical teaching aid. Covering two methods in this paper, provides researchers and teachers with the choice of using the simple historical option, the more complex historical bootstrapping method, or both.

Applications of VaR

The VaR approach to risk measurement gained a great deal of momentum following the launch of the RiskMetrics Technical document on VaR and subsequent updates (J.P. Morgan & Reuters 1996). In a banking environment, VaR has become the standard market risk measure since adoption by the Basel Committee on Banking Supervision (Bank for International Settlements, 2006) of VaR as the primary measure of market risk for determining bank capital adequacy. The appealing simplicity of the VaR concept has led to its adoption as a standard risk measure not only for financial entities involved in large scale trading operations, but also retail banks, insurance companies, institutional investors, and non-financial enterprises. In addition to the Bank for International Settlements, its use is also encouraged by the American Federal Reserve Bank and the Securities and Exchange Commission. There is extensive literature coverage on VaR. Examples include Beder (1995), Jorion (1996; 2001), J.P. Morgan & Reuters (1994; 1996), Duffie and Pan (1997), Pritsker (1997) and Stambaugh (1996), as well as comprehensive discussion of VaR by more than seventy recognised authors in the VaR Modeling Handbook and the VaR Implementation Handbook (Gregoriou 2009a; 2009b). In the financial literature, VaR is most often applied to share price analysis but has many other applications, for example exchange rates (Mittnik

2000), interest rates (Ferreira & Lopez 2005), portfolio optimisation (Campbell, Husiman & Koedijk 2001), hedge funds (Bali, Gokcan & Liang 2007), credit risk (Allen & Powell 2009; Gupton, Finger & Bhatia 1997) and energy markets (Cabedo & Moya 2003; Chiu, Chang & Lai 2010).

Information Required to Calculate VaR

There are five essential pieces of information required: Amount of exposure, risk factor or factors, risk horizon, data series of the risk factors, and the level of confidence. The first piece of essential information is the amount of exposure, which is the mark-to-market dollar value of the asset or portfolio.

A risk factor is the source of variability of the market value of the asset or portfolio such as a price (e.g., share returns), a reference rate (e.g., changes in an interest or foreign exchange rate) or an index value (e.g., volatility of a market index, such as Standard & Poor's ASX 200). The variability of this risk factor can be handily described by a histogram in the nonparametric methods or a probability distribution function in the parametric methods.

The length of the risk period has to exceed the time needed for an orderly liquidation of the asset or portfolio. Following this vein of thought, the risk period of a non-liquid asset (e.g., a piece of land) far exceeds that of a liquid asset (a share), and the risk period of a thinly traded share far exceeds that of a blue chip stock.

For each risk factor, a sufficiently long data series is required to determine the variability or randomness of the risk factor. There is no single ideal length, as the optimal length depends on the objectives of the researcher or investor. A daily trader would use a shorter length, whereas an investor interested in long term returns whould incorporate enough observations to be representative of all states of the portfolio, encompassing both upturn and downturn economic conditions.

The frequency of the data series collected preferably equals the risk horizon. If one is interested in how much one could possibly lose over the next day, one should collect daily data for the risk factor, and so on. Nevertheless, there are practitioners who prefer having frequency shorter than the risk horizon to maximise the amount of information contained in the data.

Whilst, in practice VaR is calculated at a range of confidence levels from 90 - 99.9 percent depending on how confident the user wants to be about the results, the level is most commonly set at either 95 or 99 percent (see Hedricks, 1996). For purposes of illustrating VaR calculation in this paper, the 95 percent level, in line with RiskMetrics (J.P. Morgan & Reuters 1994; 1996) , is used.

Nonparametric Calculation of VaR

Relative to the parametric approach, the nonparametric approach has the major attraction of avoiding the danger of misspecifying the distribution(s) of the risk factor(s), which could lead to under or over estimating VaR. This is especially true when recent history includes periods of non-normal trading, such as financial crises, where the distribution would likely to be left skewed with non-continuous jumps in returns. In these circumstances, the historical probability density function (PDF) is unlikely to follow a parametric distribution. This gives the nonparametric approach a role in calculating VaR measures in an era of frequent financial disturbance. The two nonparametric methods to be discussed allow us to draw conclusions about the characteristics of a population strictly from the sample at hand, rather than by making perhaps unrealistic assumptions about the population.

We discuss two nonparametric methods in this section: Historical method and historical bootstrap method. The historical method is the simplest of all methods of calculating VaR. The historical bootstrap method is a step up from the more basic historical method using the concept of bootstrapping to efficiently estimate the statistics of the underlying unknown population distribution of the risk factor. The statistical procedure of bootstrapping has its merit in providing a good approximation of the PDF of the population of the risk factor, which is not usually normally distributed, provided it is done properly.

Any historical method, by construct, assumes the PDF(s) of the risk factor(s) from which future values are drawn at the end of the risk horizon is identical to the PDF(s) over some specific historical time horizon. That is, the key to any historical method is assuming that history repeats itself, hence its name. In practice, it is impossible to choose the relevant historical time horizon with entire accuracy. The exercise is somehow arbitrary because nobody has the prevision of future events. This means the inclusion of a longer data series is preferable to a shorter data series as the former contains more information and covers more scenarios. There are authors (e.g., Hendricks, 1996) who argue that the use of shorter historical time series better mimics the potential PDF of the risk factor if there is no structural change. If the historical data series collected is a good representation of the near future, the two methods have a good track record. So the performance of the two methods hinges greatly on whether history is a good indicator of the near future or not.

Under the nonparametric assumption, VaR is calculated using only the sample statistics of past asset returns. In the context of market risk, it involves using the historical returns of the asset(s) in question.

The Teaching Study

To illustrate the use of the two methods, the teaching study uses four shares listed on the New York Stock Exchange. These four listed shares are all from different industries and are Coca Cola, Bank of America, Boeing, and Verizon Communication. Coca Cola is used to demonstrate the calculation of VaR of a single asset using the two nonparametric methods. The four shares are then combined to illustrate portfolio VaR. To simplify our discussion, we assume that there is only one underlying risk factor: the price of the share.

In this exercise, to demonstrate VaR for a single asset, an investor's exposure is $1M ($V$) worth of Coca Cola shares at time t (any trading day after 3 August 2010). The risk factor is returns on the price of the share (p), risk horizon is one trading day, historical time series is 10 years from 4 August 2001 to 3 August 2010 (a total of 2,513 observations of adjusted closing price), and the level of confidence (α) is 95 per cent. The question of interest is: In 95 out of a 100 times, what would be the worst daily loss one could experience by holding $1M Coca Cola shares?

To demonstrate VaR for a portfolio of assets (using the same historical period, number of observations, risk horizon and α used for the single asset above) the teaching study assumes an investor has a total portfolio exposure (V) of $5M comprising 20 percent Coca Cola ($1M), 30 percent Bank of America ($1.5M), 30 percent Boeing ($1.5M) and 20 percent Verizon ($1M).

Historical Method for a Single Asset

This section describes how to use Excel 2007 to calculate the $(1 - \alpha)$-per-cent VaR value, as well as how to graphically display VaR by plotting a histogram for the historical returns and inserting a $(1 - \alpha)$-per-cent VaR line. Appendix 1 provides Excel screenshots which include details of all formulas.

Assume (as in our teaching study) the frequency of the historical time series matches that of the risk horizon so there is no need for time aggregation. Let there be n observations in the historical data price series, which yields $n - 1$ returns. To obtain the $(1 - \alpha)$-per-cent VaR return, use the Excel function PERCENTILE(return series,$1 - \alpha$). Alternatively, one can multiply the $n - 1$ returns by $1 - \alpha$ to get the number of the lower $(1 - \alpha)$ percent observation, then apply the Excel function SMALL(return series, $(n - 1)(1 - \alpha)$) to arrive at the $(1 - \alpha)$-per-cent VaR return. This return is then applied to the initial value V to arrive at the $(1 - \alpha)$-per-cent VaR. For simplicity, brokerage fees have been omitted from the calculation.

To plot the histogram (see Table 2 and Figure 2 in Appendix 1 for further details), calculate an appropriate bin size such that there are at least 20 to 30 bins. Calculate the frequency of each bin using the FREQUENCY(return series, upper bins) function. Construct the frequency distribution, and plot the histogram using a column chart. When the $(1 - \alpha)$ percent VaR line is inserted into the histogram; this turns the chart from a column chart to a combination chart that contains both a column chart and a scatter chart. The procedure for inserting a dynamic $(1 - \alpha)$ percent VaR line which will respond to various values of α, is as follows: Copy the table of data (see Cells I23:J24 in Table 2 of Appendix 1) related to the $(1 - \alpha)$ percent VaR line to be incorporated into the histogram, select the histogram diagram and paste special (see Figure 1). In the paste special dialog box of the Home Ribbon of Excel 2007, select New Series, Values (Y) in Columns, and Category (X Labels) in First Column.

Figure 1
Paste Special Dialog Box
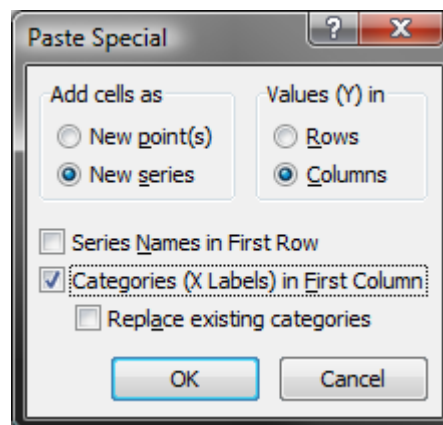
Select the new series and change the chart type from Column series to XY series (specify it as the Scatter with Straight Lines subtype). Excel displays two secondary value axes in the chart. For the new secondary vertical axis, format it from 0 (minimum value) to 1 (maximum value). For the new secondary horizontal axis, format it to match the primary horizontal axis. If the line does not appears on the chart, select the chart and check the "Select Data" entry and re-edit the x-axis and the y-axis entry. For the application of this method to our numerical example, see Appendix 1.

Historical Bootstrap Method for a Single Asset

We can improve the performance of the historical method by bootstrapping, which involves resampling the data with replacement many times in order to generate an empirical estimate of the entire sampling distribution of a statistic. Babu and Singh (1983) showed that the bootstrap sampling distribution resembles that of the population as the number of resamples increases to infinity.

The historical bootstrap method retains the same model structure as the historical data series. It treats the historical data series as if it is the population, and randomly selects historical observations which are then resampled *m* times using the scenario sampling technique taking each observation as a scenario. Since the historical data series contain *n* observations, the *m* bootstrap samples are also of size *n*. Resampling mimics the random process of the system.

We calculate the mean, standard deviation, and 5 percent VaR of each bootstrap sample and then plot the distribution of the *m* statistics. The more *m* bootstrap samples generated, the closer the averages of the three statistics of the samples would be to those of the history data series obtained by the historical method. Excel can handle a large number of resamples (we have used *m* = 1,000 in our example shown in Appendix 2, but only show five resamples on the screenshot in Table 3), but as *m* increases to high numbers such as 1,000, processing times are slowed. It is recommended that for teaching purpose a much smaller number of resamples are used to illustrate the process.

One technical aspect of Excel has to be taken care of before performing the bootstrap exercise. It is to reset the way Excel calculates. Go to Excel Options, select Formulas on the left hand side panel in the Excel Option dialog box. Under the heading of Calculation Options, select the option "Automatic except for data tables" and enable iterative calculation by setting "Maximum Iterations" = 1. Without this crucial step, the bootstrap exercise will run forever as the Excel program keeps recalculating itself. The bootstrap samples can be "recalculated" by pressing the "F9" key once.

Teachers should note that if they wish to use screenshots in the classroom, Excel has a useful screenshot function for displaying row and column numbers. First click on Page Setup under the Page Layout Ribbon, then Sheet, then tick Row and Column Headings, then OK. Highlight the Excel section to be copied, click the arrow below the Paste icon on the Home ribbon, then select As Picture, Copy as Picture, As Shown when Printed, then OK. Then just paste into the required document.

Multiple Asset Portfolio

Historical Portfolio VaR is a relatively simple calculation (as compared to the parametric approach where correlation between the assets is measured and matrix multiplication is used to calculate variance-covariance). The daily total portfolio returns are obtained by calculating the daily weighted average of the returns for each stock as shown in Table 4. As correlations across assets are naturally embedded in the historical time series, they require no separate estimation. The required confidence level (the 95 percentile worst return in our case) is then applied to the weighted average returns in Column C of Table 4, using exactly the same methodology as previously outlined for Coca Cola, and this figure is the portfolio VaR.

A potential problem with the historical approach is that the relative weightings of assets in the portfolio could have been changing over the risk period. To overcome this, a method called historical simulation is used (Choudhry 2004). Suppose a portfolio comprises shares A and B. Where their weights do not vary over the risk period, the end value could be easily calculated. If the weights vary over the risk period, then the initial value of the

portfolio has to be recalculated. Assume at the end of the risk period we have respectively *a* of the portfolio in share *A* and (1-*a*) in share *B*, we will re-weight all the historical prices according to the weights at the end of the risk period. This method makes sense as an investor is interested in potential risk based on their current weighted holdings of a portfolio, as opposed to any prior portfolio mix.

The multiple asset historical bootstrap method works in exactly the same manner as previously described for a single asset, except that the bootstrap samples are derived from the portfolio returns (weighted average returns as calculated in Table 4) as opposed to the returns for a single asset. These bootstrap calculations are shown in Table 5.

Teaching Study Results

Using 10 years of data, 5 percent daily VaR for Coca Cola was calculated in Table 1 to be -2.20 percent ($21,979) of the portfolio value of $1m. This means that an investor investing $1m in this asset could be 95 percent confident of not losing more than this amount on a given future day, based on history repeating itself. The multiple asset approach in Table 4 showed 5 percent VaR to be -2.63 percent ($131,334) of the portfolio of $5m. The historical bootstrapping method finds daily 5 percent VaR's for both Coca Cola and for the four share portfolio to be very similar to the VaR's calculated for the historical method.

Conclusion

The study has demonstrated how two nonparametric VaR calculations can be easily generated using Excel, a readily available modelling package. Excel handles considerable quantities of observations, multiple shares and large resampling numbers, all within one Excel workbook. The methods demonstrated in this paper can be used by researchers or investors to build their own nonparametric VaR models. The techniques shown and teaching study can be used in teaching students the building of VaR models. This could be in the classroom, an elab, or as an assignment whereby students can use the methods shown in this paper to build their own models for a given asset or portfolio.

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Appendices

The Appendices capture six screenshots (Tables 1, 2, 3, 4, 5, and Figure 2) from the teachingstudy spreadsheet to illustrate how the historical and historical bootstrap methods were calculated. Note that for illustrative purposes the tables only show the first few observations, but all 2,512 observations have been included in the determination of VaR.

Appendix 1: INDIVIDUAL ASSET VaR CALCULATION

Table 1
Individual Asset Historical VaR

	A	B	C	D	E	F	G	H	I	J	K
1											
2		**Coca Cola: 5% VaR by Historical Method**									
3											
4		**Data**			**VaR Analysis**						
5			Daily								
6		Obs	Returns		Numer of obs		2,512	Formula: =COUNT(cocadaily1)			
7		1	-1.71%		Min daily return		-12.33%	Formula: =MIN(cocadaily1)			
8		2	1.31%		Max daily return		8.11%	Formula: =MAX(cocadaily1)			
9		3	-3.36%		Average daily return		-0.004%	Formula: =AVERAGE(cocadaily1)			
10		4	0.10%		Range		20.45%	Formula: =G8-G7			
11		5	1.13%		Confidence level		95.00%	Value = 0.95			
12		6	1.93%		Lower 5% of obs		125.00	Formula: =ROUNDDOWN((1-G11)*G6,0)			
13		7	0.10%		5% VaR daily return		-2.20%	Formula: =SMALL(cocadaily1,G12)			
14		8	-1.52%								
15		9	-1.95%		Last opening price		$50.00	Value = 50			
16		10	-1.15%		5% VaR opening price		$48.90	Formula: =G15*(1-ABS(G13))			
17		11	0.42%		Initial value		$1,000,000	Value = 1000000			
18		12	0.10%		5% VaR		-$21,978.91	Formula: =G17*G13			

This screenshot shows the Excel functions used to calculate the 5 percent VaR value, as shown in Cell G18. For the functions applied, see Column H of the spreadsheet. Note that "cocadaily1" in the formulas is the name given to the historical data series (C7:C2519). For brevity we only show the first 12 returns. From our calculation with $V = \$1M$, risk horizon = 1 day, $n = 2,512$, $\alpha = 95$ percent, and $p = \$50$, we find that the 5 percentile return is the 125th lowest observation, 5 percent VaR daily return = -2.20 percent, 5 percent VaR price = $48.90, and the 5 percent VaR value = -$21,978.91.

Table 2
VaR Histogram

	E	F	G	H	I	J	K	L	M	N	O	P
20	**Charting**											
21												
22	Bin size		0.30%		5% daily VaR line			Cell(G22) - user to select appropriate bin size				
23	Number of bins		80		-2.20%	0		Cell(G23) - user to select appropriate number of bins				
24	Lowest bin	-13.00%			-2.20%	1		Cell(G24)=ROUNDUP(MIN(cocadaily1),2)				
25	Highest bin	11.00%						Cell(G25)=G24+(G23*G22)				
26								Cells (I23 and I24)=5%VaR as obtained from table 1, Cell(G13)				
27		Lower	Upper	Absolute	Relative	X-axis						
28	0	bin	bin	freq	freq	label						
29	1	-13.00%	-12.70%	0	0.0000	-12.9%		Cell(F29)=G24				
30	2	-12.70%	-12.40%	0	0.0000	-12.6%		Cell(G29)=F29+G22				
31	3	-12.40%	-12.10%	1	0.0004	-12.3%		Cell(H30)=Frequency(cocadaily1,G30:G108)				
32	4	-12.10%	-11.80%	0	0.0000	-12.0%		Cell (I30)=H30/SUM(H:H)				
33	5	-11.80%	-11.50%	0	0.0000	-11.7%		Cell(J30)=AVERAGE(F30:G30)				
34	6	-11.50%	-11.20%	0	0.0000	-11.4%						
35	7	-11.20%	-10.90%	0	0.0000	-11.1%						
36	8	-10.90%	-10.60%	1	0.0004	-10.8%						
37	9	-10.60%	-10.30%	0	0.0000	-10.5%						
38	10	-10.30%	-10.00%	0	0.0000	-10.2%						
39	11	-10.00%	-9.70%	0	0.0000	-9.9%						
40	12	-9.70%	-9.40%	0	0.0000	-9.6%						

This screenshot shows the workings for the histogram. For plotting the histogram, user needs to select the number of bins required (we use 80, but for brevity show only the first 12 bins below). This should start from a point which includes the lowest return (in our case the lowest return per Table 1 is -12.33 percent, so we have started from a return of -13 percent). We have chosen each bin size to be at intervals of 0.3 percent, which based on 80 bins gives a maximum point on the histogram of 11 percent. This covers our maximum return of 8.11 percent per Table 1. The frequencies are calculated as per the formulas in Column L. The relative frequencies are used to plot a bar chart (histogram as per figure 2). A 5 percent daily VaR line is inserted according to the method described in the main body.

Figure 2
Historical one-day 5 percent VaR, Coca Cola
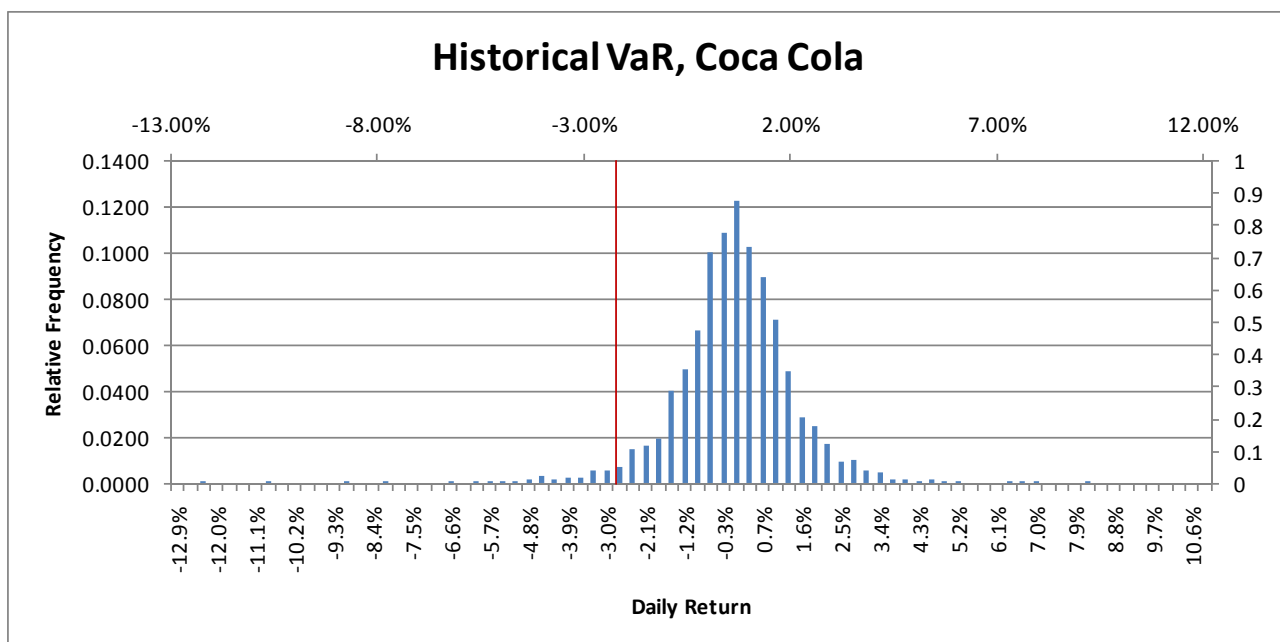
This shows the histogram of the Coca Cola returns to Coca Cola share and its corresponding 5 percent VaR line using the historical method. Construction is as discussed in Table 2 and in the main body.

Table 3
Individual Asset Historical Bootstrap Method

	E	F	G	H	ALP	ALQ	ALR	ALS	ALT	ALU
2	**Coca Cola: 5% VaR by Historical Bootstrap Method**									
3										
4										
5										
6		BS1	BS2	BS3	BS999	BS1000				
7	1	-2.27%	0.94%	2.15%	-0.07%	0.11%				
8	2	0.53%	-0.96%	-2.80%	1.84%	3.40%				
9	3	-2.31%	0.00%	-0.50%	-0.61%	1.97%				
10	4	-1.34%	-0.35%	0.09%	-0.82%	-1.61%				
11	5	-0.80%	2.60%	-0.43%	3.55%	2.78%				
12	6	1.97%	1.39%	-0.27%	1.84%	-0.28%				
2513	2507	0.73%	-0.27%	-0.45%	-0.50%	1.28%				
2514	2508	-0.70%	0.94%	-0.69%	0.09%	1.49%				
2515	2509	2.05%	-0.96%	-0.43%	1.28%	1.56%				
2516	2510	0.55%	0.50%	0.85%	0.10%	0.55%				
2517	2511	-0.67%	1.10%	-0.85%	-1.19%	0.84%				
2518	2512	0.80%	0.54%	-0.32%	1.40%	0.89%	CellALQ2518=VLOOKUP(RANDBETWEEN(1,2512),B:C,2,FALSE)			
2519							(note columns B:C are as per table 1)			
2520										
2521		BS1	BS2	BS3	BS999	BS1000				
2522	Number of obs	2512	2512	2512	2512	2512				
2523	Min daily return	-12.33%	-12.33%	-12.33%	-12.33%	-10.63%	Cell(ALQ2523)=Min(ALQ$7:ALQ$2518)			
2524	Max daily return	8.11%	8.11%	8.11%	8.11%	6.87%	Cell(ALQ2524)=MAX(ALQ$7:ALQ$2518)			
2525	Average daily return	0.01%	0.01%	0.00%	-0.03%	-0.02%	Cell(ALQ2525)=AVERAGE(ALQ$7:ALQ$2518)			
2526	Range	20.44%	20.44%	20.44%	20.44%	17.50%	Cell(ALQ2526)=ALQ2524-ALQ2523			
2527	Confidence level	95%	95%	95%	95%	95%	Cell(ALQ2527)=95%			
2528	Lower 5% of obs	125	125	125	125	125	Cell(ALQ2528)=ROUNDDOWN((1-ALQ2527)*ALQ2522,0)			
2529	5% VaR daily return	-2.18%	-2.20%	-2.14%	-2.25%	-2.16%	Cell(ALQ2529)=SMALL(ALQ$7:ALQ$2518,ALQ2528)			
2530			Bootstrap Method							
2531			(Averages, Max, and Min of Rows 2520:2529)	Comparison to Historical Method (per Table 3)						
2532	Number of obs		2512	2512						
2533	Min daily return		-12.33%	-12.33%						
2534	Max daily return		8.11%	8.11%						
2535	Average daily return		0.02%	-0.01%						
2536	Range		20.44%	20.44%						
2537	Confidence level		95.00%	95.00%						
2538	Lower 5% of obs		125.00	125.00						
2539	5% VaR daily return		-2.20%	-2.20%						

The following screenshot shows the workings for taking 1,000 bootstrap samples. The 2,512 historical observations of returns are treated as 2,512 scenarios. When resampling, they are randomly selected by using the RANDBETWEEN(1, 2512) function and the corresponding returns are captured by the VLOOKUP( ) function, see the formula printed in Cell ALR2518 in the screenshot. Once the 1,000 bootstrap samples are done, descriptive statistics and 5 percentVaR for each sample is calculated (see rows 2522 to 2529). From the individual 1,000 bootstrap samples, the overall 5 percent VaR is measured as the average of the samples (see Column G, Rows 2532:2539). A comparison to the historical method is provided in Column ALQ. Indeed, the two sets of figures come very close.

Appendix 2: MULTIPLE ASSET Var calculation

Table 4
Four Share Portfolio Historical Approach

	A	B	C	D	E	F	G	H	I	J
1										
2		**Four Shares Portfolio: Weighted Average Returns**								
3				20.00%	30.00%	30.00%	20.00%			
4		**Data**		Daily Returns	Daily Returns	Daily Returns	Daily Returns		**VaR Analysis: Four Shares Portfolio**	
5			Weighted	Returns	Returns	Returns	Returns			
6		Obs	Average	Coca Cola	B of America	Boeing	Verizon		Numer of obs	2,512
7		1	0.90%	-1.71%	4.00%	-0.13%	0.40%		Min daily return	-16.21%
8		2	-1.22%	1.31%	-0.96%	-0.64%	-5.02%		Max daily return	12.70%
9		3	-1.95%	-3.36%	-0.24%	0.00%	-6.03%		Average daily return	-0.01%
10		4	-0.84%	0.10%	2.38%	-2.21%	-4.54%		Range	28.91%
11		5	1.41%	1.13%	0.35%	1.95%	2.45%		Confidence level	95.00%
12		6	0.96%	1.93%	0.23%	2.29%	-0.91%		Lower 5% of obs	125.00
13		7	0.37%	0.10%	2.42%	-2.16%	1.36%		5% VaR daily return	-2.63%
14		8	-1.51%	-1.52%	-0.80%	-3.54%	0.45%			
15		9	-1.76%	-1.95%	-1.51%	-2.16%	-1.36%			
16		10	-0.88%	-1.15%	-0.47%	0.14%	-2.77%			
17		11	1.98%	0.42%	0.82%	0.00%	8.23%		Initial Value	$ 5,000,000
18		12	1.05%	0.10%	0.23%	3.87%	-1.01%		5% VaR	-$ 131,334
19		13	-0.17%	-1.47%	1.04%	2.33%	-4.45%			
20		14	0.36%	-1.07%	-1.27%	2.77%	0.61%		**Note:**	
21		15	1.24%	-1.62%	-2.35%	6.27%	1.94%		Row 3 = weightings	
22		16	0.23%	-0.22%	-1.56%	-0.23%	4.06%		Cell C7 =SUMPRODUCT(D$3:G$3,D7:G7)	
23		17	-0.26%	-0.88%	0.36%	-0.82%	0.28%		Copy formula all the way down Column C	

Daily returns for each of the four shares are calculated in the same manner as for Coca Cola in Table 1.
Weighted average returns are calculated as per the note in Cells I20:I 23 below. All other formulas are as per
Table 1, except they are applied to the weighted average in Column C as opposed to cocadaily1.

Table 5
Four Share Portfolio Historical Bootstrap Method

	D	E	F	G	H	ALP	ALQ	ALR	ALS
2		**Four Shares Portfolio: 5% VaR by Historical Bootstrap Method**							
3									
4									
5									
6			BS1	BS2	BS3	BS999	BS1000		
7		1	-16.21%	0.47%	1.62%	-0.95%	-0.67%		
8		2	-0.57%	0.45%	-0.02%	-0.30%	0.26%		
9		3	0.51%	0.36%	-0.38%	-1.56%	-3.70%		
10		4	0.84%	0.05%	0.08%	-0.37%	0.11%		
11		5	1.01%	1.33%	-1.25%	-0.36%	-0.40%		
12		6	0.29%	0.24%	-0.08%	1.02%	2.43%		
2513		2507	-1.05%	9.18%	-0.43%	-1.36%	-0.17%		
2514		2508	-2.80%	-0.02%	8.62%	-0.80%	1.02%		
2515		2509	1.19%	-3.80%	1.37%	1.14%	-1.34%		
2516		2510	-2.68%	9.17%	0.10%	-1.37%	-5.32%		
2517		2511	-0.13%	-1.25%	-0.19%	-0.41%	-0.87%		
2518		2512	-4.82%	-1.47%	0.60%	1.64%	0.06%		
2519			BS1	BS2	BS3	BS999	BS1000	Bootstrap Method (Averages, Max and Min of Rows 2520:2527)	Comparison to Historical Method (per Table 4)
2520		Number of obs	2512	2512	2512	2512	2512	2512	2512
2521		Min daily return	-16.21%	-15.69%	-16.21%	-16.21%	-15.69%	-16.21%	-16.21%
2522		Max daily return	12.70%	12.70%	12.70%	10.27%	12.70%	12.70%	12.70%
2523		Average daily return	0.03%	0.02%	0.00%	-0.03%	0.00%	0.01%	-0.01%
2524		Range	28.91%	28.40%	28.91%	26.48%	28.40%	28.91%	28.91%
2525		Confidence level	95%	95%	95%	95%	95%	95.00%	95.00%
2526		Lower 5% of obs	125	125	125	125	125	125.00	125.00
2527		5% VaR daily return	-2.63%	-2.61%	-2.72%	-2.68%	-2.61%	-2.63%	-2.63%
2528									

Bootstrap samples (1000) are calculated in the same manner as for Coca Cola in Table 3, except they are applied to the weighted average returns (as calculated in Column C of Table 4) as opposed to cocadaily1. The outcomes for the Bootstrap method are shown in Column ALR and compared to the Historical Method outcomes (as per Table 4) in Column ALS.